Chestnuts Primary School

Written Methods of Calculation

A booklet for parents

Help your child with mathematics

March 2016

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You want to help your child with her maths homework, but you; wonder what she means when she says that she has been using the *grid method* all morning, are not sure what *complementary addition* has to do with subtraction and are bewildered by *partitioning*.

Help could be at hand. This booklet will try to explain the written methods of recording addition, subtraction, multiplication and division used in school and the progression within these methods.

The principle behind these methods is that they build on mental methods of calculation to develop an understanding of what is really happening and not just blindly following a standard method. This may have been the way many of us were taught. The methods work, but did you really understand why at the time?

What may then be a surprise is that we aim to reach these standard methods but with a clearer understanding of what is being done and a better idea of whether the answer seems right.

Progression within the methods should not be seen as directly matched to a particular year, but as with reading, a continuum to move along as and when ready. However, The National Curriculum sets clear expectations and the assumption that most children will broadly progress at the same rate. A brief guide is shown on the next page. (These are end of year expectations.)

While in Foundation and early Key Stage 1, most children will not be expected to use paper and pencil procedures for calculation, although they may make jottings and certainly record their calculations with number sentences. Their experience of these operations will be a mixture of practical, oral and mental work.

We place great importance on providing children with clear representations of what is happening in a calculation through the use of *concrete* (real) materials such as Tens frames, Dienes blocks (base ten material), Cuisenaire rods, place value counters, place value (arrow) cards, multi-link cubes and all manner of real objects. Moving to *pictorial* representations which will still give an image to represent the calculation but without the manipulative qualities of real objects leads into a more *abstract* representation of the written method.

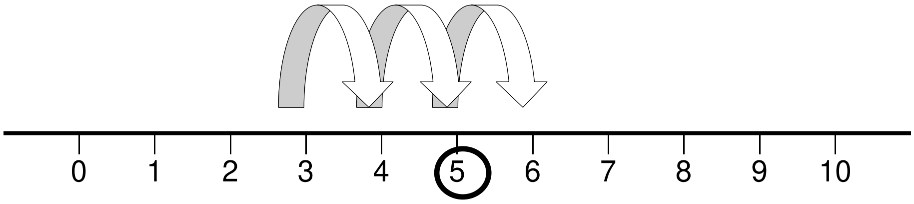
Although many people will be more familiar with the term 'Units', the term 'Ones' is used throughout this booklet (following guidance in the National Curriculum). These terms should be seen as the same but the use of 'Ones' encouraged.

Year	Addition	Subtraction	Multiplication	Division
1	Add one-digit and two-digit numbers to 20, including zero.	Subtract one-digit and two-digit numbers to 20, including zero.	Calculating using concrete objects, pictorial representations, arrays	Calculating using concrete objects and pictorial representations.
2	Add numbers using concrete objects, pictorial representations including: • a two-digit number and ones • a two-digit number and tens • two two-digit numbers • adding three one-digit numbers	Subtract numbers using concrete objects, pictorial representations including: • a two-digit number and ones • a two-digit number and tens • two two-digit numbers • adding three one-digit numbers	calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs	calculate mathematical statements for division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
3	Add numbers with up to three digits, using formal written methods of columnar addition.	Subtract numbers with up to three digits, using formal written methods of columnar subtraction.	Progress towards writing and calculating mathematical statements for multiplication using known multiplication tables, including for two-digit numbers times one-digit numbers using columnar methods.	Progress towards writing and calculating mathematical statements for division using known multiplication tables, including for two-digit numbers times one-digit numbers using columnar methods.
4	Add numbers with up to 4 digits using the formal written methods of columnar addition.	Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction.	Multiply two-digit and three-digit numbers by a one-digit number using the formal written method of short multiplication.	Pupils practise to become fluent in the formal written method of short division with exact answers.
5	Add whole numbers with more than 4 digits, including using formal written methods.	Subtract whole numbers with more than 4 digits, including using formal written methods.	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.	Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders.
6	Solve addition multi-step problems in contexts, deciding which operations and methods to use and why.	Solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why.	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.	Divide numbers up to 4 digits by a two-digit whole number using the formal written method and interpret remainders.

Use in conjunction with the following written methods and the notes on the previous page.

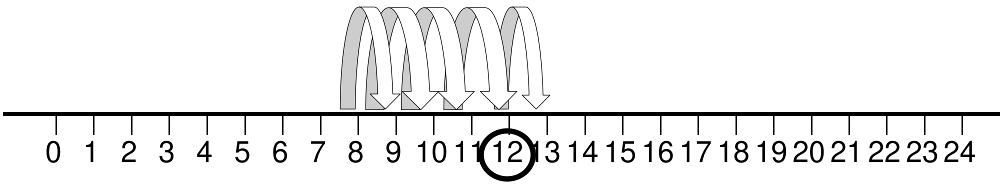
Written Strategies
Children should be able to explain what they are doing with the use of practical apparatus before they are encouraged to record anything. Informal jottings should also be encouraged.

Addition – Partitioning
Counting on Ones + Ones
5 + 3 = 8

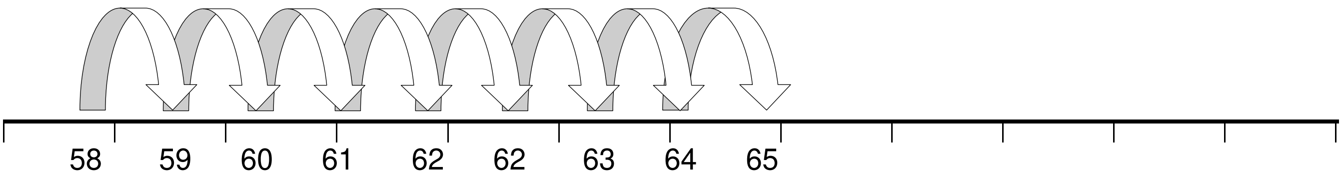


Able to count on from 5 and explain that adding 3 takes them to 8 because it is 3 hops on from 5.

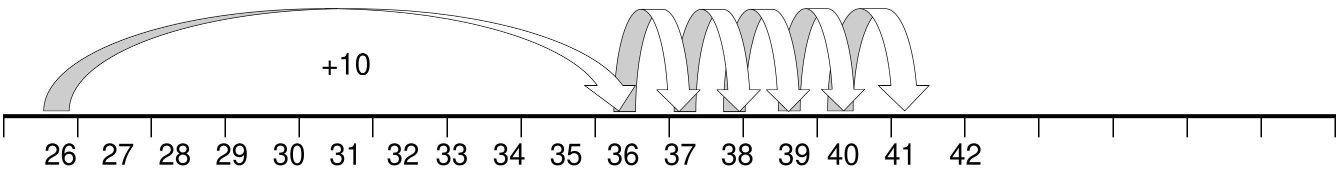
Counting on Tens and Ones + Ones
12 + 5 = 17



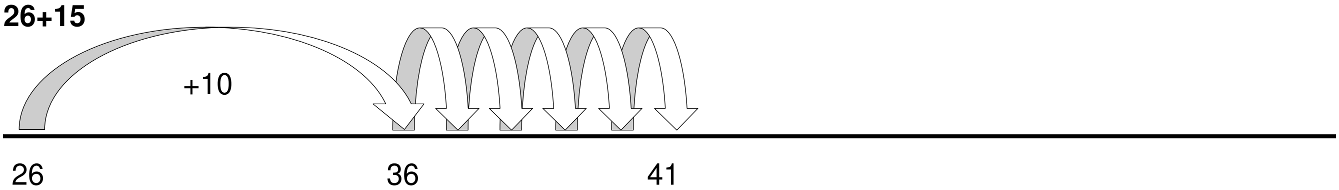
Extend to larger numbers and crossing a tens boundary Tens and Ones + Ones
56 + 8 = 64



Extend to Tens and Ones + Tens Ones
Beginning at the largest number and partitioning the other number
26 + 15



Children progress from published number lines to drawing their own to show understanding.
26+15



Discourage recording numbers under all the hops as matching the number to the point where the hop touches the line can lead to mistakes.

Adding Tens and Ones to Tens and Ones through partitioning.

32	+	24			(no exchange or carrying)
30	+	2			
20	+	4			
50	+	6	=	56	

35	+	27			(with exchange or carrying)
30	+	5			
20	+	7			
50	+	12	=	62	

Could initially partition the second number to ease mental addition

50 + 10 + 2 = 62

A useful transition between the informal numberline and formal column addition stressing the importance of partition into tens and ones.

Vertical addition no exchange or carrying (expanded method)

23 + 42

	2	3
+	4	2
		5
	6	0
	6	5

Beginning with the least significant digit

The expanded method will rarely involve carrying and, as such, is a natural progression from the above method of partitioning.

Vertical addition with exchange or carrying

	4	3	5	
+	3	6	7	
		1	2	(5 ones + 7 ones)
		9	0	(3 tens + 6 tens)
	7	0	0	(4 hundreds + 3 hundreds)
	8	0	2	
	1			

Adding the least significant digit first to include exchange or carrying up to Hundreds, Tens and Ones
Developing to adding

Hundreds, Tens and Ones + Tens and Ones
Hundreds, Tens and Ones + Hundreds, Tens and Ones
Thousands, Hundreds, Tens and Ones + Hundreds, Tens and Ones
Thousands, Hundreds, Tens and Ones + Thousands, Hundreds, Tens and Ones

Vertical compact addition – exchanging/carrying, in one column only

	4	3	5
+	3	2	7
	7	6	2
		1	

Adding 5 ones and 7 ones will give 12. The 2 (ones) is placed under the line in the ones column and the 1 ten placed below the next space in the tens column to be added to the total of the tens.

Vertical addition – exchanging or carrying in 2 columns

	4	3	5
+	3	9	7
	8	3	2
	1	1	

Vertical addition – exchange or carrying

Hundreds, Tens and Ones + Hundreds, Tens and Ones
Thousands, Hundreds, Tens and Ones + Hundreds, Tens and Ones
Thousands, Hundreds, Tens and Ones + Thousands, Hundreds, Tens and Ones

Continued with and without exchange or carrying into decimals, money and measures including time.

	2	5	7	·	0	6
		3	2	·	5	
+	6	7	1	·	9	8
	9	6	1	·	5	4
	1	1	1		1	

Written Strategies

Children should be able to explain what they are doing with the use of practical apparatus before they are encouraged to record anything. Informal jottings should also be encouraged.

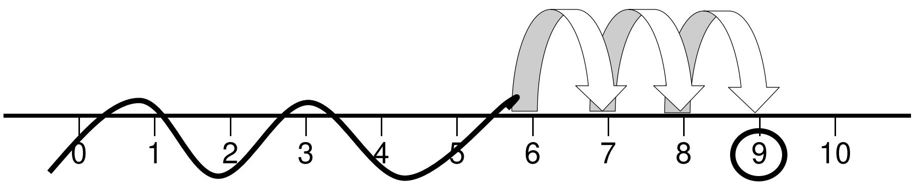
Subtraction – Complementary Addition

The language of subtraction (taking away) is immediately familiar to even the youngest children who, through play, will be more regularly taking objects away, and using that vocabulary, than they would be talking of adding objects together. Subtraction, therefore, should be a more familiar concept but it is this familiarity that can be a problem for children learning to subtract.

Taking away with a line of sorting animals or cubes in a small group is good early practise but then forming these into a line and taking the objects away from the ***start*** of the line, rather than the end will lead more easily into complementary addition – finding the difference.

Ones take away Ones

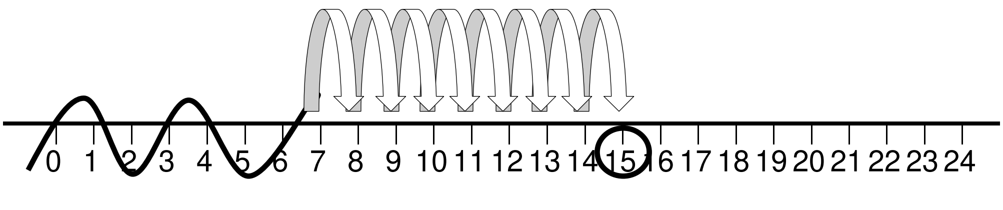
9 – 6 = 3



Cross out the numbers from 0 to (including) the number to be taken away and then count on. Circling the number to stop at (9) can help but care is needed to ensure this end number is also counted.

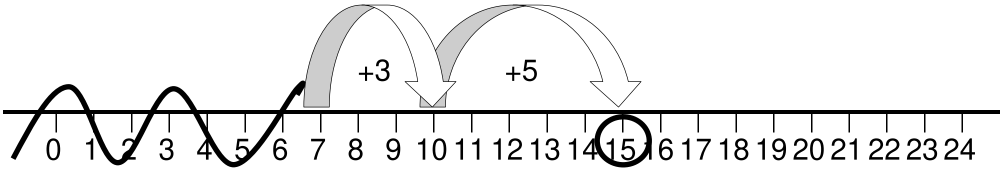
Tens and Ones subtract Ones (Following the same format; counting on in ones.)

15 – 7 = 8



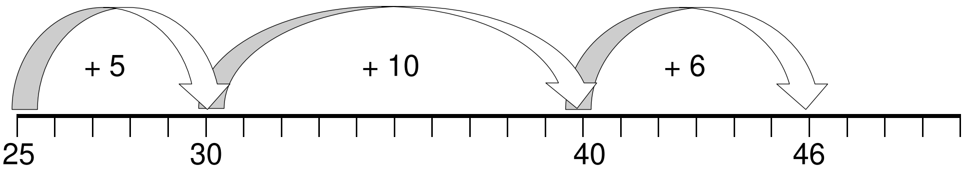
Tens and Ones subtract Ones (Counting on to the next ten – using complement knowledge)

15 – 7 = 8



Extend to Tens and Ones - Tens and Ones (Counting on to the next ten – using complement knowledge)

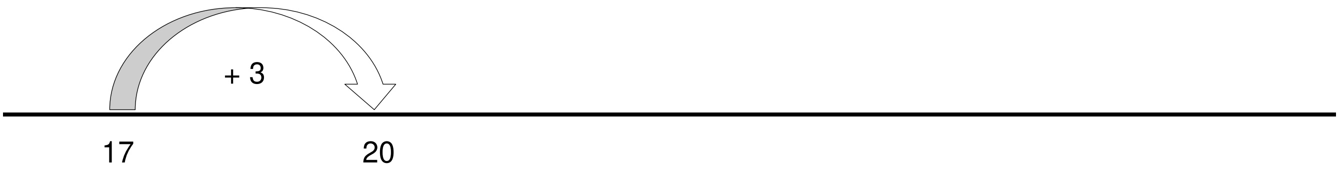
46 – 25 = 21



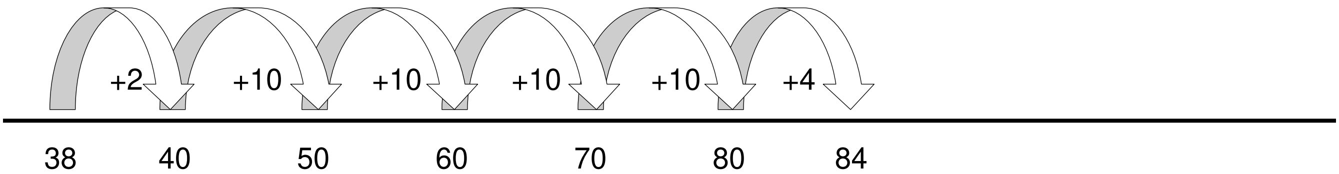
Children progress from using published number lines to drawing their own empty numberlines. The benefit being the ability to accommodate any calculation.

Tens and Ones subtract Tens and Ones

20 – 17 = 3



84 – 38 = 46



At this point, most children will have an understanding of subtraction as difference and be able to apply this method to larger numbers and even decimals.

The introduction of a formal column method of subtraction can be introduced when children fully understand subtraction as finding the difference. Examples 'without borrowing' should be quickly followed by those that include 'borrowing' to ensure it is seen that sometimes taking away of some columns cannot be done. (Clearly this can be done, but would involve negative numbers which can be left to a later date.)

Column method (without borrowing)

$$\begin{array}{rcc} & 6 & 8 \\ - & 3 & 6 \\ \hline & 3 & 2 \end{array}$$

Care should be taken to ensure children do not over generalise before learning how to 'borrow'.

Popular misconception:

$$\begin{array}{rcc} & 6 & 6 \\ - & 3 & 8 \\ \hline & 3 & 2 \end{array}$$

Taking away the ones column, can't do 6 take away 8 or will do 8 take away 6 and write the answer as 2.

Standard method (decomposition) Column method (with borrowing)

$$\begin{array}{rcc} & {}^{5}\not{6} & {}^{1}2 \\ - & 3 & 6 \\ \hline & 2 & 6 \end{array}$$

- Start with the ones column. 2 take away 6 can't be done, so borrow 10 from the tens column (reducing 60 to 50) and add it to the 2 (12).
- 12 take away 6 is 6. Write 6 in the ones column under the answer line.
- 5 (50) take away 3 (30) is 2 (20).
- Write 2 in the tens column under the answer line.

Not having anything to borrow in the next column can cause problems, thinking through carefully can be solved

Part 1

$$\begin{array}{rccc} & {}^{7}\not{8} & {}^{10}0 & 2 \\ - & 3 & 6 & 5 \\ \hline \end{array}$$

- Start with the ones column. 2 take away 5 can't be done, so borrow 10 from the tens column. But there are no tens in the tens column. Borrow 100 from the hundreds column (reducing 800 to 700) and add it to the 0 (100). It should be seen at this point that the number (802) has not been changed just altered to 700+100+2.

Part 2

$$\begin{array}{rccc} & {}^{7}\not{8} & {}^{9\not{10}}0 & {}^{1}2 \\ - & 3 & 6 & 5 \\ \hline & 4 & 3 & 7 \end{array}$$

- 2 take away 5 still can't be done, so borrow 10 from the tens column (reducing 100 to 90) and add it to the 2 (12).
- 12 take away 5 is 7. Write 7 in the ones column under the answer line.
- 9 (90) take away 6 (60) is 3 (30).
- Write 3 in the tens column under the answer line.
- 7 (700) take away 3 (300) is 4 (400).
- Write 4 in the hundreds column under the answer line.

Extending this method to include decimals

	H	T	O		$^{1}/_{10}$	$^{1}/_{100}$
	^{8}~~9~~	$^{1\ 6}$~~7~~	$^{1\ 1}$~~2~~	·	$^{1\ 7}$~~8~~	^{1}3
-	4	8	5	·	9	5
	4	8	6	·	8	8

- Start with the $^{1}/_{100}$ column. 3 take away 5 can't be done, so borrow $^{1}/_{10}$ from the $^{1}/_{10}$ column (reducing 0·8 to 0·7) and add it to 0·03 (0·13).
- 0·13 take away 0·05 is 0·08. Write 8 in the $^{1}/_{10}$ column under the answer line.
- Continue with the $^{1}/_{10}$ column. 7 (0·7) take away 9 (0·9) can't be done, so borrow 1 from the ones column (reducing 2 to 1) and add it to 0·7 (1·7).
- 1·7 take away 0·9 is 0·08. Write 8 in the $^{1}/_{10}$ column under the answer line.
- Continue with the ones column. 1 take away 5 can't be done, so borrow 10 from the tens column (reducing 70 to 60) and add it to 1 (11).
- 11 take away 5 is 6. Write 6 in the ones column under the answer line.
- Continue with the tens column. 6 (60) take away 8 (80) can't be done, so borrow 100 from the hundreds column (reducing 900 to 800) and add it to 60 (160).
- 16 (160) take away 8 (80) is 8 (80). Write 8 in the tens column under the answer line.
- Complete the calculation with the hundreds column. 8 (800) take away 4 (400) is 4 (400). Write 4 in the hundreds column under the answer line.
- A final estimate, does it look about right and even checking by adding 486·88 to 485·95 (the inverse) ensures the correct answer.

Written Strategies
Children should be able to explain what they are doing with the use of practical apparatus before they are encouraged to record anything. Informal jottings should also be encouraged.

Multiplication – Grid Method
Arrays
(Teach ÷ alongside × using number lines and arrays)

Children need to be able to group objects and explain what they are doing clearly.
Be able to count on in equal groups
Record as arrays and be able to give related facts.

x	x	x
x	x	x
x	x	x
x	x	x

3 groups of 4 = 12

4 groups of 3 = 12

x	x	x	x
x	x	x	x
x	x	x	x

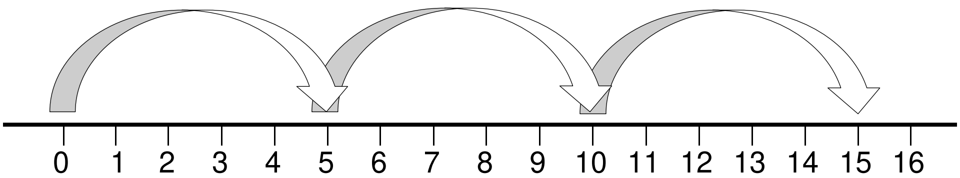
Understand inverse:

- How many groups of 3 can be made from 12?
- How many groups of 4 can be made from 12?
- How do you know?

<u>**Repeated addition**</u>
3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 3 x 5

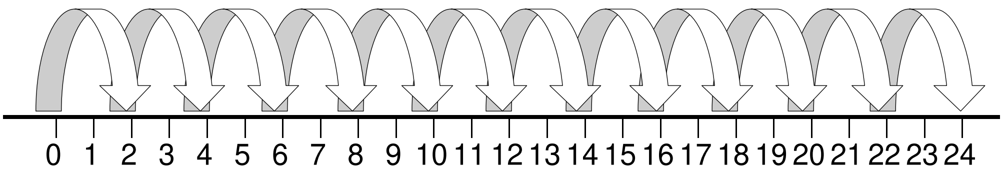
Repeated addition can be shown easily on a number line:
3 x 5 = 5 + 5 + 5

Ones multiplied by Ones
3 x 5 = 15



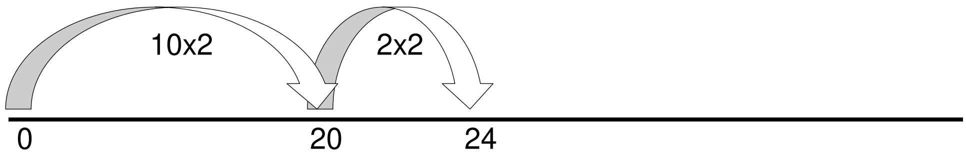
Children should be understand the commutatively, that 3x5 is the same as 5x3 (and note the similarity with addition).

Tens and Ones multiplied by Ones
12 x 2 =24



Children should recognise that while this method will continue to work, it is not efficient and open to errors the larger the numbers and more jumps involved.

Using an empty numberline to show Tens and Ones x Ones by partitioning
12 x 2 =24



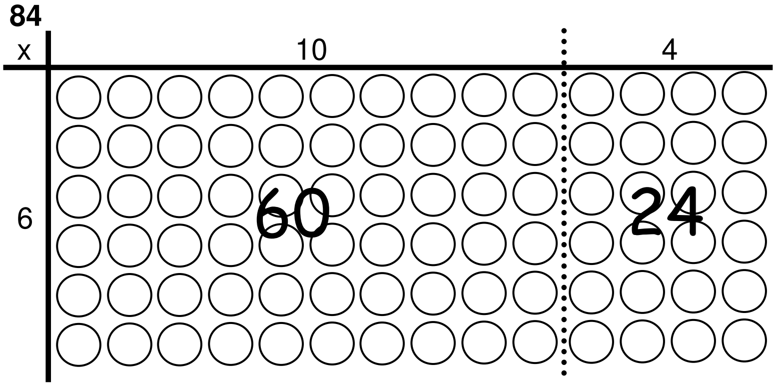
Partition without number line.
23 x 2 = 46

20 x 2 = 40
3 x 2 = 6

40 + 6 = 46

Can be modelled to show clearly what is happening, but moving to a simple grid method directly from an empty numberline would not be a problem for many children.

Grid Method as an array
14 x 6 = 84



A good model to link arrays and calculation and as an image for the children to visualise when using the grid method.

Grid Method - Tens and Ones x Ones (by partition of the numbers)
14 x 6 = 84

x	10	4
6	60	24

60 + 24 = 84

Initially, 24 could be partitioned to make the addition easier
60 + 20 + 4 = 84

Multiplying a 3-digit number by a 1-digit number would be the next step.

Grid method Tens and Ones x Tens and Ones
27 x 43 = 1161

x	20	7
40	800	280
3	60	21

800 + 280 = 1080
60 + 21 = 81

1080 + 81 = 1161

Extend to decimals
23.62 x 12.5

x	20	3·0	0·6	0·02
10	200	30·0	6·0	0·20
2	40	6·0	1·2	0·04
0·5	10	1·5	0·3	0·01

	236·2
	47·24
	11·81
	295·25
carries	1 1

Whilst the grid method is very clear and will work for even quite complex numbers, it does involve adding the individual ‘answers’ and, more importantly, is not classed as a formal method of calculation. Children should be moving to Long or Expanded multiplication after mastering the Grid method.

Long multiplication (expanded) Tens and Ones x Ones
32 x 4 = 128

A formal written method

	32	
x	4	
	8	(4x2)
	120	(4x 30)
	128	

Recording each step at the side, will initially help keep track of what is being done.

Begin by multiplying the least significant digit first.

Multiplying a 3-digit number by a 1-digit number would be the next step.

Long multiplication (expanded) Tens and Ones x Tens and Ones
27 x 43

A formal written method

	27	
x	43	
	21	(3x 7)
	60	(3x20)
	280	(40x 7)
	800	(40x20)
	1161	
carries	1	

Extend to larger numbers and decimals to two places.

Short multiplication will appeal to those children who have mastered short division and are looking for a more efficient method. There is, however, no great need to move beyond the expanded method for 2-digit (and larger) by 2-digit numbers.

Short multiplication (standard method)
27 x 3 = 81

A formal written method

	2	7
x		3
	8	1
	2	

3x7=21. Write 1 in the ones column under the answer line and ‘carry’ the 2 tens, writing below the tens column.

Short multiplication (standard method)
27 x 43 = 1161

A formal written method

		2	7
x		4	3
		8	1
		2	
1	0	8	0
	2		
1	1	6	1
	1		

Recording ‘carrying’ (the 2 tens from 21 (3x7)) below the tens column should be done with care and not mistakenly added to the 8 (80) and 8 (80) to get the final answer.

Short multiplication (standard method)
3427 x 43

		3	4	2	7
x				4	3
	1	0	2	8	1
		1		2	
1	3	7	0	8	0
	1	1	2		
1	4	7	3	6	1
			1		

A formal written method

Written Strategies
Children should be able to explain what they are doing with the use of practical apparatus before they are encouraged to record anything. Informal jottings should also be encouraged.

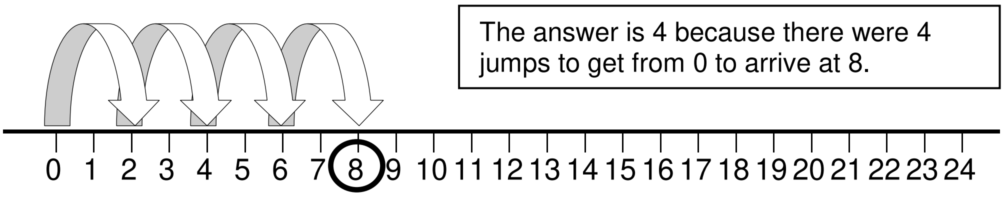
Division

Children will experience division from an early age; the idea of sharing, of it being fair (everyone to have the same) and this is ideal to continue in Key Stage 1 from Foundation.
Division needs to be given in context, wherever possible, to ensure understanding of sharing and later grouping.

Practical examples of sharing.

I have 8 sweets to share between 2 of us, how many will each of us get?

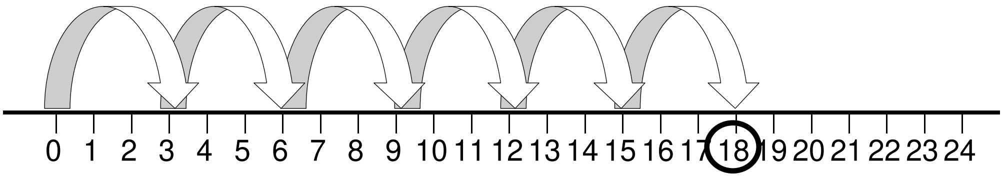
Divide Ones by Ones on a numberline counting on in equal groups
8 ÷ 2 = 4



0 + (4 groups of 2) = 8 (Repeated addition)

Making connections between the numbers landed on and multiplication tables is important.

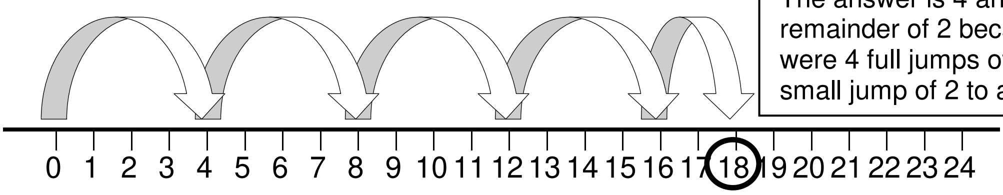
Divide Tens and Ones by Ones on a numberline counting on in equal groups
18 ÷ 3 = 6



0 + (6 groups of 3) = 18 (Repeated addition)

Divide Tens and Ones by Ones on a numberline counting on in equal groups with a remainder
18 ÷ 4 = 4^{r2}

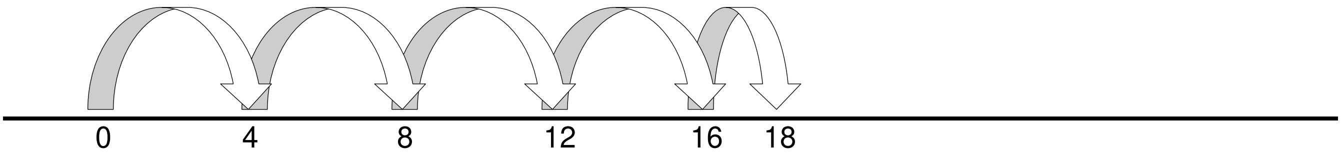
The answer is 4 and a remainder of 2 because there were 4 full jumps of 4 and a small jump of 2 to arrive at 18.



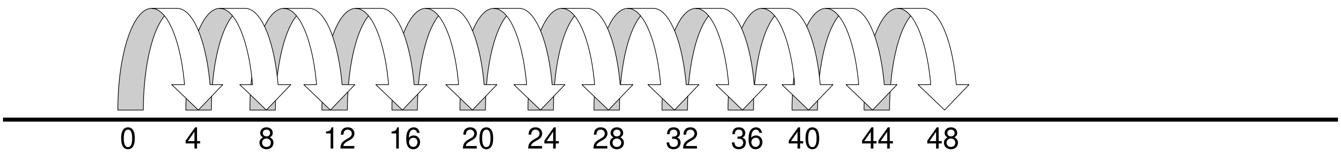
0 + (4 groups of 4) + 2 left over = 18(Repeated addition)Show as 4 ^{r}2 or 4 $^{2}/_{4}$ or 4 ½

Children progress from using published number lines to drawing their own empty numberlines. The benefit being the ability to accommodate any calculation.

Divide Tens and Ones by Ones on an empty numberline counting on in equal groups with a remainder
18 ÷ 4 = 4^{r2}

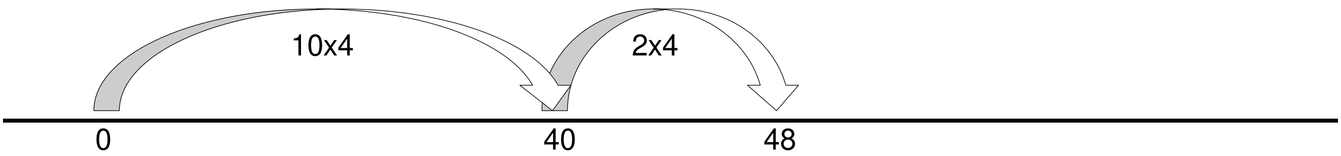


Divide Tens and Ones by Ones on an empty numberline counting on in equal groups
48 ÷ 4 = 12



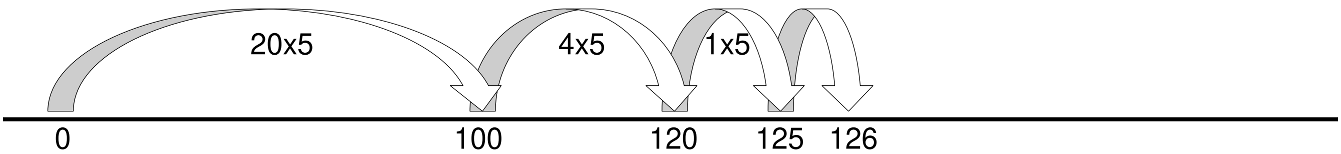
Recognise that this can take a long time with more opportunity for simple error, so a more efficient method would be to jump in groups in a similar fashion to multiplication at this stage.

48 ÷ 4 = 12



Divide Tens and Ones by Ones on an empty numberline counting on in equal groups

126 ÷ 5 = 25^{r1}



Becoming more efficient by taking multiples of 10 times (20x5, 4x5, 1x5 and a remainder of 1)

Taking this method any further, moves into vertical ‘chunking’. This causes too many children too many problems, often not recognising larger multiples of the divisor and, therefore creating a large number of subtractions where errors frequently occur. Moreover, ‘chunking’ is not easily extended to division of decimals.

Division – Long division

Whilst different, long division is not dissimilar to ‘chunking’ it differs in that it will consider the most significant digits first rather than the whole number as in ‘chunking’.
*It should be worth considering whether making the jump to **Short Division** would be easier.*

Divide Tens and Ones by Ones using long division
126 ÷ 5 = 25 ^{r1}

			2	5	r	1	(Thinking not generally recorded)
5	)	1	2	$^{2}6$			12÷5=2 r2; 2x5=10
	-	1	0	↓			Bring the 6 down
			2	6			5 into 26=5 r1; 5x5=25
		-	2	5			
				1			Remainder 1

Divide Tens and Ones by Tens and Ones using long division
384 ÷ 16 = 24

				2	4	(Thinking not generally recorded)
1	6	)	3	8	$^{6}4$	38÷16=2 r6; 2x16=32
		-	3	2	↓	Bring the 4 down
				6	4	16 into 64=4; 4x16=64
			-	6	4	
					0	No remainder

Division – Short division *(Bus stop)* supported by jottings

Use of jottings of the first few numbers in the multiplication table of the divisor can significantly increase the effectiveness of this method.

Divide Tens and Ones by Ones using long division
126 ÷ 5 = 25 ^{r1}

			2	5	r	1
5	)	1	2	$^{2}6$		

- There are no groups of 5 in 1.
- Take the 1 and the next digit, 2.
- There are 2 groups of 5 in 12 with a remainder of 2 (write 2 above the 2 and the 2 beside the next digit, 6).
- There are 5 groups of 5 in 26 with a remainder of 1 (write 5 above the $^{2}6$ and r1 next the 25).

			2	5	·	2
5	)	1	2	$^{2}6$	·	$^{1}0$

- The remainder can be extended to be shown as a decimal

Divide Tens and Ones by Tens and Ones using long division

384 ÷ 16 = 24

				2	4			
1	6	)	3	8	$^{6}4$			

- There are no groups of 16 in 3.
- Take the 3 and the next digit, 8.
- There are 2 groups of 16 in 38 with a remainder of 6 (write 2 above the 8 and the 6 beside the next digit, 4).
- There are 4 groups of 16 in 64 (write 4 above the $^{6}4$).

16
32
48
64

(the first few multiples of 16)

Extended into decimals

377.92 ÷ 16 = 23.62

				2	3	·	6	2
1	6	)	3	7	$^{5}7$	·	$^{9}9$	$^{3}2$

16
32
48
64
80
96

(the first few multiples of 16)

“Dad taught me how he does multiplication,” is not something we would encourage until the child has mastered multiplication and can understand how this other method is different and why it works. We would encourage exploring different methods when this will develop understanding rather than confuse – please think carefully and if unsure, stick to the methods in this book.

The whole basis of using these methods is that the child should understand and be confident at any one stage before moving to the next. Rushing ahead will not help and may well create gaps in understanding which will impact on the ability to understand subsequent methods, variation in the presentation of a problem, applying the method to solve worded problems and other (connected) areas of maths such as fractions and division.

We hope that this booklet is of help; that you understand the written methods we use for calculation and the progression within these methods. However, if you do have any questions, please ask your child’s teacher, they will be happy to explain.